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RAN-2003000201030034

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F.Y.B.Sc. (Sem. - I) (ATKT) Examination

March - 2023

MTH - 102 Mathematics - II

Time: 1 Hour] **[Total Marks: 50**

સૂચના : / Instructions

(૧)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **F.Y.B.Sc. (Sem. - I) (ATKT)**

Name of the Subject :

☛ **MTH - 102 Mathematics - II**

Subject Code No.: **2003000201030034**

Seat No.:

Student's Signature

(2) આ પ્રશ્નપત્રમાં (૧) થી (૧૬) પ્રશ્નના ૧ ગુણ અને (૧૭) થી (૩૩) પ્રશ્નના ૨ ગુણ છે.
Question (1) to (16) carry ONE mark and (17) to (33) carry TWO marks.

(3) દરેક પ્રશ્નનો ફક્ત એક જ સાચો ઉત્તર છે.
There is only ONE correct answer for each question.

(4) પ્રચલિત સંકેતોને અનુસરો.
Follow usual symbols.

(5) પરીક્ષાનો સમય ૧ કલાક નો છે.
The EXAM is of 1 hour duration.

O.M.R. Sheet ભરવા અંગેની અગત્યની સૂચનાઓ આપેલ
O.M.R. Sheetની પાછળ છાપેલ છે.
Important instructions to fillup O.M.R. Sheet
are given on back side of the provided O.M.R. Sheet.

SECTION- A / વિભાગ - A

(Question number 1 to 16 each is of 1 mark)

(પ્રશ્ન ક્રમાંક 1 થી 16, દરેકનો 1 ગુણ છે.)

1. $f(x)\ \frac{1}{1+x^2}$ is ________ function in $(-\infty, 0)$

(A) Increasing (B) Constant

(C) Decreasing (D) None of these

$f(x)\ \frac{1}{1+x^2}$ વિધેય $(-\infty, 0)$ માં ________ છે.

(A) વધતું (B) અચળ

(C) ઘટતું (D) આ પૈકી એક પણ નહિ

2. A real valued function f is continuous on $[a, b]$ and differentiable in $[a, b]$ then there exists $\lambda \in (a,b)$ such that $f'(\lambda) =$ ________.

(A) $\frac{f(b)-f(a)}{b-a}$ (B) $\frac{f(a)-f(b)}{b-a}$

(C) 0 (D) None of these

જો વાસ્તવિક વિધેય f એ $[a, b]$ માં સતત અને $[a, b]$ માં વિકલનીય હોય તો $\lambda \in (a, b)$ એવા મળે જે જેથી $f'(\lambda) =$ ________.

(A) $\frac{f(b)-f(a)}{b-a}$ (B) $\frac{f(a)-f(b)}{b-a}$

(C) 0 (D) આ પૈકી એક પણ નહિ

3. A real valued function f is continuous on $[0, x]$ where $x > 0$ and differentiable in $(0, x)$ then $f(x) - f(0) =$ ________ Where $\theta \in (0, 1)$.

(A) $f'(\theta x)$ (B) $xf'(\theta)$

(C) $xf'(\theta x)$ (D) $\theta f'(\theta x)$

જો વાસ્તવિક વિધેય f એ $(0, x)$ જ્યા $x > 0$ માં સતત અને $(0, x)$ માં વિકલનીય હોય તો $f(x) - f(0) =$ ________ જ્યાં $\theta \in (0, 1)$.

(A) $f'(\theta x)$ (B) $xf'(\theta)$

(C) $xf'(\theta x)$ (D) $\theta f'(\theta x)$

4. Curve $y = e^{5x}$ is ________.

(A) Concave downward (B) Concave upward

(C) Convex upwards (D) None of these

વક્ર $y = e^{5x}$ ________ છે.

(A) અધ:અંતમુખ (B) ઉધર્વ–અંતર્મુખ

(C) ઉધર્વ–બહિર્મુખ (D) આ પૈકી એક પણ નહિ

5. The point of inflexion of $y = 3x^5 - 40x^3 + 3x - 20$ is ________.

(A) $x = 0$ (B) $x = 2$

(C) $x = -2$ (D) All of these

$y = 3x^5 - 40x^3 + 3x - 20$ નું વક્રતા પરીવૃતિ બિંદુ ________ છે.

(A) $x = 0$ (B) $x = 2$

(C) $x = -2$ (D) આ પૈકી બધા

6. Vertical asymptote of the curve $y = \frac{2x-3}{x^2-3x+2}$ are ________.

વક્ર $y = \frac{2x-3}{x^2-3x+2}$ ના લંબક અનંત સ્પર્શકો ________છે.

(A) $x = 1$ and $x = -2$ (B) $x = -1$ and $x = 2$

(C) $x = 1$ and $x = 2$ (D) $x = -1$ and $x = -2$

7. The curvature of the curve $y = \sin x$ at the point $\left(\frac{\pi}{2}, 1\right)$ is ________.

વક્ર $y = \sin x$ ની $\left(\frac{\pi}{2}, 1\right)$ બિંદુએ વક્રતા ________ છે.

(A) 1 (B) 2

(C) 3 (D) 4

8. $\int_0^{\pi/2} \sin^7 x dx =$ ________.

(A) $-\frac{35}{16}$ (B) $\frac{16}{35}$

(C) $-\frac{16}{35}$ (D) $\frac{35}{16}$

9. Justify that $\int_0^{\pi/2} \sin^{10} x dx = \frac{63}{512}$ is true?

(A) Yes (B) No

(C) Can't say anything (D) Undefined

વિધાન $\int_0^{\pi/2} \sin^{10} x dx = \frac{63}{512}$ સાંચુ છે?

(A) હા (B) ના

(C) કશું જ ન કહેવાય (D) અવ્યાખ્યાયિત

10. $\int_0^{\pi/2} \sin^4 x \cos^3 x \, dx =$ ________.

(A) $\frac{2}{35}$ (B) $\frac{2}{35}\pi$

(C) $\frac{35}{2}$ (D) $\frac{35}{2}\pi$

11. $\int \cos ec^n xdx =$ ________.

(A) $\frac{\cot x \cos ec^{n-2}x}{n-1} + \frac{n-2}{n-1} \int \cos ec^{n-2} xdx$

(B) $\frac{\cot x \cos ec^{n-2}x}{n-1} - \frac{n-2}{n-1} \int \cos ec^{n-2} xdx$

(C) $\frac{-\cot x \cos ec^{n-2}x}{n-1} - \frac{n-2}{n-1} \int \cos ec^{n-2} xdx$

(D) $\frac{-\cot x \cos ec^{n-2}x}{n-1} + \frac{n-2}{n-1} \int \cos ec^{n-2} xdx$

12. If $y = \frac{1}{(3x+2)^6}$ then $y_n =$ ________.

જો $y = \frac{1}{(3x+2)^6}$ હોય તો $y_n =$ ________.

(A) $\frac{(-1)^n (n-5)!2^n}{5!(3x+2)^{n+6}}$ (B) $\frac{(-1)^n (n-5)!3^n}{5!(3x+2)^{n+6}}$

(C) $\frac{(-1)^{n+1} (n-5)!2^n}{5!(3x+2)^{n+6}}$ (D) $\frac{(-1)^{n+1} (n-5)!3^n}{5!(3x+2)^{n+6}}$

13. If $y = \log x$ then $y_n =$ ________.

જો $y = \log x$ હોય તો $y_n =$ ________.

(A) $\frac{(-1)^n (n-1)}{x^n}$ (B) $\frac{(-1)^n (n-1)!}{x^n}$

(C) $\frac{(-1)^{n+1} (n-1)}{x^n}$ (D) $\frac{(-1)^{n-1} (n-1)!}{x^n}$

14. If $y = a^{mx+k}$ then $y_n =$ ________.

જો $y = a^{mx+k}$ હોય તો $y_n =$ ________.

A) $m^n (\log a)^n a^{mx+k}$ (B) $m^n (\log a) a^{mx+k}$

(C) $m (\log a)^n a^{mx+k}$ (D) $m (\log a) a^{mx+k}$

15. If $y = x^4 + 3x^3 + 2x^2 + x + 1$ then $y_4 =$ ________.

જો $y = x^4 + 3x^3 + 2x^2 + x + 1$ હોય તો $y_4 =$ ________.

(A) 4! (B) 4

(C) 3! (D) 3

16. For real valued function f,

$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!} f''(a) + \frac{h^3}{3!} f'''(a) + ... + \frac{h^{n-1}}{(n-1)!} f^{n-1}$

$(a) + R; 0 < \theta < 1$ is called

________.

(A) Taylor's expansion (B) Maclaurin's expansion

(C) A and B both (D) None of these

વાસ્તવિક વિધેય f માટે

$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!} f''(a) + \frac{h^3}{3!} f'''(a) + ... + \frac{h^{n-1}}{(n-1)!} f^{n-1}$

$(a) + R; 0 < \theta < 1$ ને ________કહેવાય છે.

(A) ટેલર નું વિસ્તરણ (B) મેકલોરીન વિસ્તરણ

(C) A અને B બંને (D) આ પૈકી એક પણ નહિ

SECTION - B / વિભાગ - B

(Question number 17 to 33 each is of 2 marks)

(પ્રશ્ન ક્રમાંક 17 થી 33, દરેકના 2 ગુણો છે.)

17. For which value of λ, function $f(x) = \log x; x \in [1,e]$ satisfy

$f'(\lambda) = \dfrac{f(e) - f(1)}{e-1}$?

જો $f(x) = \log x; x \in [1,e]$ તો (λ) ની કઈ કિમંત માટે $f'(\lambda) = \dfrac{f(e) - f(1)}{e-1}$ થાય?

(A) $e - 1$ (B) $\dfrac{1}{e-1}$

(C) $e + 1$ (D) $\dfrac{1}{e+1}$

18. The curvature of the curve $y = \log_e x$ at point (1, 0) is ________.

વક્ર $y = \log_e x$ ની (1, 0) બિંદુએ વક્રતા ________ છે.

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{\sqrt{2}}$

(C) $2\sqrt{2}$ (D) $\dfrac{1}{2\sqrt{2}}$

19. Curve $y = \cos x; x \in (-2\pi, 2\pi)$ becomes concave upward in ________. interval.

$y = \cos x; x \in (-2\pi, 2\pi)$ વક્ર ________ અંતરાલમાં ઉર્ધ્વ અંતર્મુખ બને છે.

(A) $\left(\frac{3\pi}{2}, 2\pi\right)$ (B) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(C) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ (D) $\left(-2\pi, -\frac{3\pi}{2}\right)$

20. Asymptote of the curve $y = \frac{x^2 + 2x - 1}{x}$ is ________.

વક્ર $y = \frac{x^2 + 2x - 1}{x}$ નો અનંત સ્પર્શક ________છે.

(A) $y = 2x + 1$ (B) $y = 2x - 1$

(C) $y = x - 2$ (D) $y = x + 2$

21. The point of inflexion of $y = x^3 - 6x^2$ is ________.

$y = x^3 - 6x^2$ નું વક્રતા પરીવૃતિ બિંદુ ________છે.

(A) $(1, 2)$ (B) $(-1, 2)$

(C) $(-1, -2)$ (D) $(1, -2)$

22. $\int \tan^4 x dx =$ ________.

(A) $\frac{\tan^3 x}{3} - \tan x + x$ (B) $\frac{\tan^3 x}{3!} - \tan x + x$

(C) $-\frac{\tan^3 x}{3} + \tan x - x$ (D) $-\frac{\tan^3 x}{3!} + \tan x - x$

23. $\int_0^{\pi/4} \sin^5 2x dx =$ ________.

(A) $\frac{4}{15}\pi$ (B) $\frac{4}{15}$

(C) $\frac{15}{4}\pi$ (D) $\frac{15}{4}$

24. $\int \cos ec^4 x dx =$ ________.

(A) $-\dfrac{\cot x \cos ec^2 x}{3} + \dfrac{2}{3}\cot x$

(B) $-\dfrac{\cot x \cos ec^2 x}{3} - \dfrac{2}{3}\cot x$

(C) $\dfrac{\cot x \cos ec^2 x}{3} + \dfrac{2}{3}\cot x$

(D) $\dfrac{\cot x \cos ec^2 x}{3} - \dfrac{2}{3}\cot x$

25. $\int x^2 \sin 2x dx =$ ________

(A) $\dfrac{x^2 \cos 2x}{2} + \dfrac{x \sin 2x}{2} + \dfrac{\cos 2x}{4}$

(B) $-\dfrac{x^2 \cos 2x}{2} + \dfrac{x \sin 2x}{2} + \dfrac{\cos 2x}{4}$

(C) $\dfrac{x^2 \cos 2x}{2} - \dfrac{x \sin 2x}{2} + \dfrac{\cos 2x}{4}$

(D) $-\dfrac{x^2 \cos 2x}{2} - \dfrac{x \sin 2x}{2} + \dfrac{\cos 2x}{4}$

26. If $y = xe^x$ then $y_n =$ ________.

(A) nxe^x (B) $(x+n)e^x$

(C) ne^x (D) None of these

જો $y = xe^x$ હોય તો $y_n =$ ________.

(A) nxe^x (B) $(x+n)e^x$

(C) ne^x (D) આ પૈકી એક પણ નહિ

27. If $y = \sin kx + \cos kx$ then $y_n =$ ________.

જો $y = \sin kx + \cos kx$ હોય તો $y_n =$ ________.

(A) $k^n [1 + (-1)^n \cos 2kx]^{\frac{1}{2}}$

(B) $k^n [1 + (-1)^n \sin 2kx]^{\frac{1}{2}}$

(C) $k^n [1 + (-1)^n \sin kx]^{\frac{1}{2}}$

(D) $k^n [1 + (-1)^n \cos kx]^{\frac{1}{2}}$

28. If $y = \frac{1}{4} [e^{2x} \sin x + e^{2x} \sin 3x]$ then $y_n =$ ________.

જો $y = \frac{1}{4} [e^{2x} \sin x + e^{2x} \sin 3x]$ હોય તો $y_n =$ ________.

(A) $\frac{e^{2x}}{4}\left[5^{\frac{n}{2}} \sin\left(x + n\, tan^{-1} \frac{1}{2}\right) + 13^{\frac{n}{2}} \sin\left(3x + n\, tan^{-1} \frac{3}{2}\right)\right]$

(B) $\frac{e^{2x}}{4}\left[5^{\frac{n}{2}} \sin\left(3x + n\, tan^{-1} \frac{3}{2}\right) + 13^{\frac{n}{2}} \sin\left(x + n\, tan^{-1} \frac{1}{2}\right)\right]$

(C) $\frac{e^{2x}}{4}\left[5^{\frac{n}{2}} \sin\left(x + n\, tan^{-1} \frac{3}{2}\right) + 13^{\frac{n}{2}} \sin\left(3x + n\, tan^{-1} \frac{1}{2}\right)\right]$

(D) $\frac{e^{2x}}{4}\left[13^{\frac{n}{2}} \sin\left(x + n\, tan^{-1} \frac{3}{2}\right) + 5^{\frac{n}{2}} \sin\left(3x + n\, tan^{-1} \frac{1}{2}\right)\right]$

29. If $y = \frac{1}{(ax + b)^m}$; $ax + b \neq 0$ and $m \in N$ then $y_n =$ ________.

જો $y = \frac{1}{(ax + b)^m}$; $ax + b \neq 0$, $m \in N$ હોય તો $y_n =$ ________.

(A) $\frac{(-1)^n (m+n)! a^n}{(m-1)!(ax+b)^{m+n}}$

(B) $\frac{(-1)^n (m+n)! b^n}{(m-1)!(ax+b)^{m+n}}$

(C) $\frac{(-1)^n (m + n - 1)! a^n}{(m-1)!(ax+b)^{m+n}}$

(D) $\frac{(-1)^n (m+n-1)! b^n}{(m-1)!(ax+b)^{m+n}}$

30. If $y = \dfrac{ax}{(ax+b)}$; $ax + b \neq 0$ then y_n = ________.

જો $y = \dfrac{ax}{(ax+b)}$; $ax + b \neq 0$ હોય તો y_n = ________.

(A) $\dfrac{(-1)^n (n)! a^n b}{(ax+\mathrm{b})^{n+1}}$ (B) $\dfrac{(-1)^{n+1} (n)! a^n b}{(ax+\mathrm{b})^{n+1}}$

(C) $\dfrac{(-1)^n n! a^{n-1} b}{(ax+\mathrm{b})^{n+1}}$ (D) $\dfrac{(-1)^{n+1} n! a^{n-1} b}{(ax+\mathrm{b})^{n+1}}$

31. In ________ interval, function $f(x) = 2x^3 - 15x^2 + 36x + 1$; $x \in R$ is decreased.

(A) $[3, \infty]$ (B) $[-\infty, 2]$

(C) $[2, 3]$ (D) None of these

વિધેય $f(x) = 2x^3 - 15x^2 + 36x + 1$; $x \in R$ ________ અંતરાલમાં ઘટતું છે.

(A) $[3, \infty]$ (B) $[-\infty, 2]$

(C) $[2, 3]$ (D) આ પૈકી એક પણ નહિ

32. For the function $f(x) = e^x$; $x \in [0,1]$, according to the Lagrange's theorem's λ = ________.

વિધેય $f(x) = e^x$; $x \in [0,1]$ માટે લાગ્રાન્જના પ્રમેય અનુસાર λ = ________.

(A) $e - 1$ (B) $\dfrac{1}{e-1}$

(C) $\log\left(\dfrac{1}{e-1}\right)$ (D) $\log(e-1)$

33. Which of the following is true for $0 < a < b$?

નીચેનું કયું $0 < a < b$ માટે સત્ય છે?

(A) $\frac{a-b}{a^2+1} < \tan^{-1} a - \tan^{-1} b < \frac{a-b}{b^2+1}$

(B) $\frac{a-b}{b^2+1} < \tan^{-1} a - \tan^{-1} b < \frac{a-b}{a^2+1}$

(C) $\frac{b-a}{a^2+1} < \tan^{-1} a - \tan^{-1} b < \frac{b-a}{b^2+1}$

(D) $\frac{b-a}{b^2+1} < \tan^{-1} a - \tan^{-1} b < \frac{b-a}{a^2+1}$

SPACE FOR ROUGH WORK